FP2 Paper 6 |*adapted 2006 JAN

1. Find the set of values of x for which $\dfrac{x^2}{x-2} > 2x.$ **(Total 6 marks)**

2. (a) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 0. \qquad \textbf{(4)}$$

(b) Given that $x = 1$ and $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 1$ at $t = 0$, find the particular solution of the differential equation, giving your answer in the form $x = \mathrm{f}(t)$.

(5)

(c) Sketch the curve with equation $x = \mathrm{f}(t)$, $0 \le t \le \pi$, showing the coordinates, as multiples of π, of the points where the curve cuts the x-axis.

(4)(Total 13 marks)

3. (a) Show that the substitution $y = vx$ transforms the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x - 4y}{4x + 3y} \qquad \text{(I)}$$

into the differential equation

$$x\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{3v^2 + 8v - 3}{3v + 4} \qquad \text{(II)}. \qquad \textbf{(4)}$$

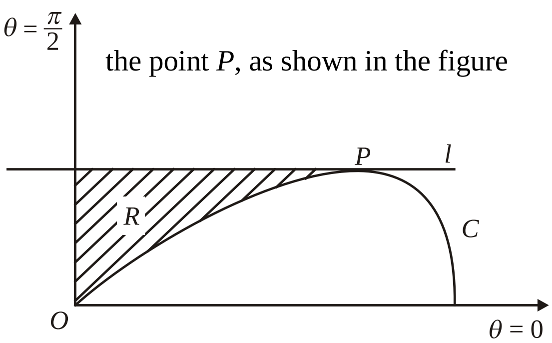
(b) By solving differential equation (II), find a general solution of differential equation (I). **(5)**

(c) Given that $y = 7$ at $x = 1$, show that the particular solution of differential equation (I) can be written as

$$(3y - x)(y + 3x) = 200.$$

(5)(Total 14 marks)

4. A curve C has polar equation $r^2 = a^2 \cos 2\theta$, $0 \le \theta \le \dfrac{\pi}{4}$.



The line l is parallel to the initial line, and l is the tangent to C at the point P, as shown in the figure above.

(a) (i) Show that, for any point on C, $r^2 \sin^2\theta$ can be expressed in terms of $\sin\theta$ and a only. (1)

(ii) Hence, using differentiation, show that the polar coordinates of P are $\left(\dfrac{a}{\sqrt{2}}, \dfrac{\pi}{6}\right)$.(6)

The shaded region R, shown in the figure above, is bounded by C, the line l and the half-line with equation $\theta = \dfrac{\pi}{2}$. (b) Show that the area of R is $\dfrac{a^2}{16}\left(3\sqrt{3} - 4\right)$.

(8)

(Total 15 marks)

5. Solve the equation $z^5 = \mathrm{i}$

giving your answers in the form $\cos\theta + \mathrm{i}\sin\theta$.

(Total 5 marks)

7.

$$(1+2x)\frac{\mathrm{d}y}{\mathrm{d}x} = x + 4y^2.$$

(a) Show that

$$(1+2x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 1 + 2(4y-1)\frac{\mathrm{d}y}{\mathrm{d}x} \qquad \boxed{1}$$

(2)

(b) Differentiate equation $\boxed{1}$ with respect to x to obtain an equation involving

$\frac{\mathrm{d}^3}{\mathrm{d}x^3}, \frac{\mathrm{d}^2y}{\mathrm{d}x^2}, \frac{\mathrm{d}y}{\mathrm{d}x}$, x and y.

(3)

Given that $y = \frac{1}{2}$ at $x = 0$,

(c) find a series solution for y, in ascending powers of x, up to and including the term in x^3.

(6)(Total 11 marks)

8. In the Argand diagram the point P represents the complex number z.

Given that $\arg\left(\frac{z-2\mathrm{i}}{z+2}\right) = \frac{\pi}{2}$,

(a) sketch the locus of P,

(4)

(b) deduce the value of $|z + 1 - \mathrm{i}|$.

(2)

The transformation T from the z-plane to the w-plane is defined by

$$w = \frac{2(1+\mathrm{i})}{z+2}, \qquad z \neq -2$$

(c) Show that the locus of P in the z-plane is mapped to part of a straight line in the w-plane, and show this in an Argand diagram.

(6)(Total 12 marks)